Computational Geometry Algorithms Library

www.cgal.org

Monique Teillaud

INRIA SOPHIA-ANTIPOLIS

Overview

- The CGAL Open Source Project
- Structure of CGAL
- The Kernel
- Numerical Robustness
- Contents of the Basic Library
- Flexibility
- Work in Progress

The CGAL Open Source Project

Goals

- Promote the research in Computational Geometry (CG)

- *"make the large body of geometric algorithms developed in the field of CG available for industrial applications"*

⇒ **robust programs**

CG Impact Task Force Report, 1996

Among the key recommendations:

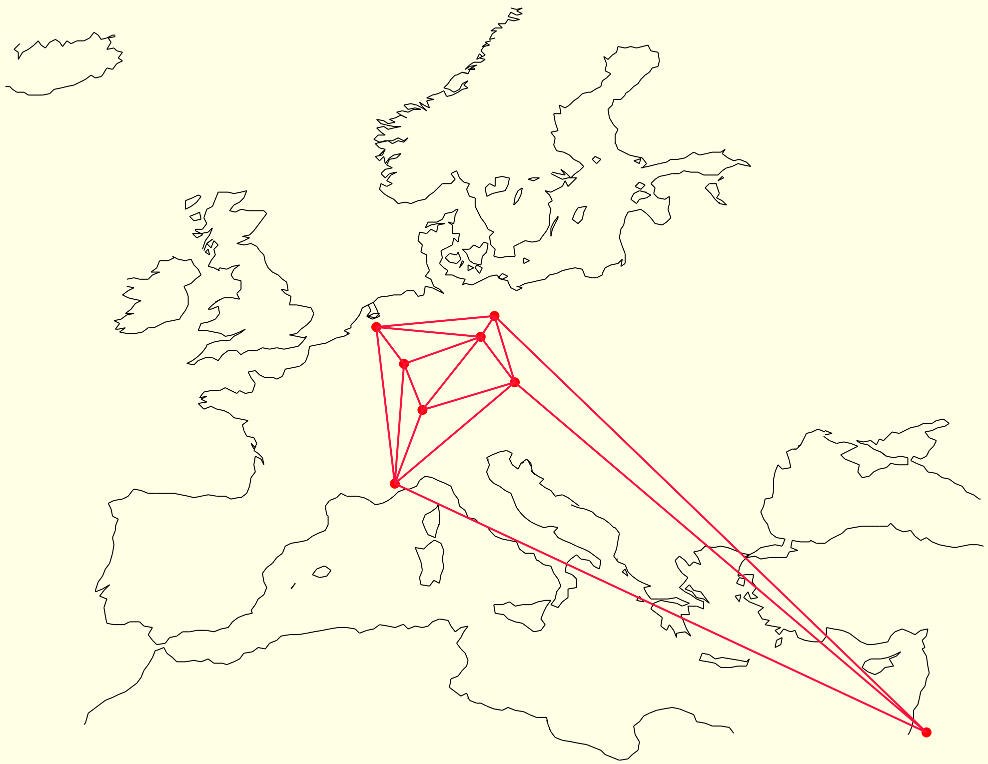
- Production and distribution of usable (and useful) geometric codes

- Reward structure for implementations in academia

History

Development started in 1995

Consortium of 8 European sites
Two ESPRIT LTR European Projects (1996-1999)



Utrecht University (XYZ Geobench)
INRIA Sophia Antipolis (C++GAL)
ETH Zürich (Plageo)
MPI Saarbrücken (LEDA)
Tel Aviv University
Freie Universität Berlin
RISC Linz
Martin-Luther-Universität Halle

- Work continued after the end of European support (1999) in several sites.

- January, 2003: **creation of Geometry Factory**

INRIA startup
sells commercial licenses, support, customized developments

- November, 2003:

Release 3.0
Open Source Project

- December, 2004: Release 3.1

License

- *kernel* under **LGPL**
- *basic library* under **QPL**
  - free use for Open Source code
  - commercial license needed otherwise

- A guarantee for CGAL users
- Allows CGAL to become a standard
- Opens CGAL for new **contributions**

CGAL in numbers

- 350.000 lines of **C++** code
- ~2000 pages manual
- release cycle of ~12 months
- CGAL 2.4: 9300 downloads (18 months)
- CGAL 3.1: 7329 downloads (9 months)
- 4000 subscribers to the announcement list (7000 for gcc)
- 800 users registered on discussion list (600 in gcc-help)
- 50 developers registered on developer list

Supported platforms

- Linux, Irix, Solaris, Windows, Mac OS X
- g++, SGI CC, SunProCC, VC7, Intel

Development process

Editorial Board created in 2001.

- responsible for the **quality** of CGAL

New packages are **reviewed**.

→ helps authors to get **credit** for their work.

CG Impact Task Force Report, 1996
Reward structure for implementations in academia

- decides about technical matters
- coordinates communication and promotion
- ...

Andreas Fabri (Geometry Factory)
Efi Fogel (Tel Aviv University)
Bernd Gärtner (ETH Zürich)
Michael Hoffmann (ETH Zürich)
Menelaos Karavelas (University of Notre Dame, USA → Greece)
Lutz Kettner (Max-Planck-Institut für Informatik)
Sylvain Pion (INRIA Sophia Antipolis)
Monique Teillaud (INRIA Sophia Antipolis)
Remco Veltkamp (Utrecht University)
Ron Wein (Tel Aviv University)
Mariette Yvinec (INRIA Sophia Antipolis)

Tools

- Own manual tools: LaTeX $\longrightarrow$ ps, pdf, html
- CVS server for version management

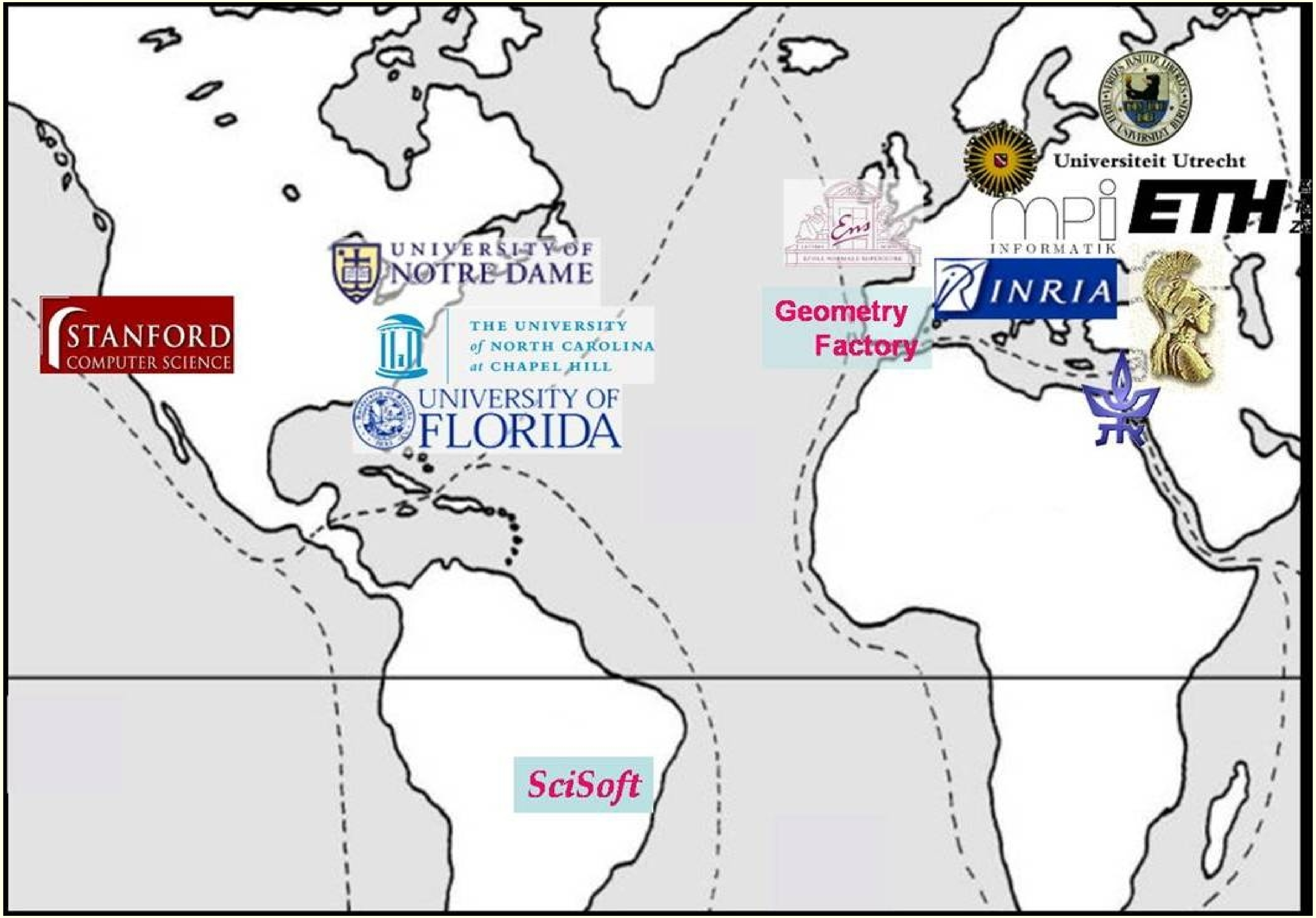
- Developer manual
- mailing list for developers
- 1-2 developers meetings per year, 1 week long

- 1 internal release per day
- Automatic **test suites** running on all supported compilers/platforms

Credit

Contributors keep their identity

- up to 3.0.1: names of authors mentioned in the Preface.
- 3.1: **Names of authors** appear at the beginning of each chapter.
  Section on history of the package at the end of each chapter, with names of all contributors.
- CGAL developers listed on the "People" web page.
- Authors publish **papers** (conferences, journals) on their packages.
- **Copyright** kept by the institution of the authors.


Universiteit Utrecht
mpi INFORMATIK
ETH
UNIVERSITY OF NOTRE DAME
STANFORD COMPUTER SCIENCE
THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL
UNIVERSITY OF FLORIDA
Ens
Geometry Factory
INRIA
SciSoft

Users

Projects using CGAL

Leonidas J. Guibas' and co-workers, Stanford University.

Tamal K. Dey's and co-workers, The Ohio State University.

Nina Amenta and co-workers, The University of Texas at Austin.

Xiangmin Jiao, University of Illinois at Urbana-Champaign.
(Surface Mesh Overlay)

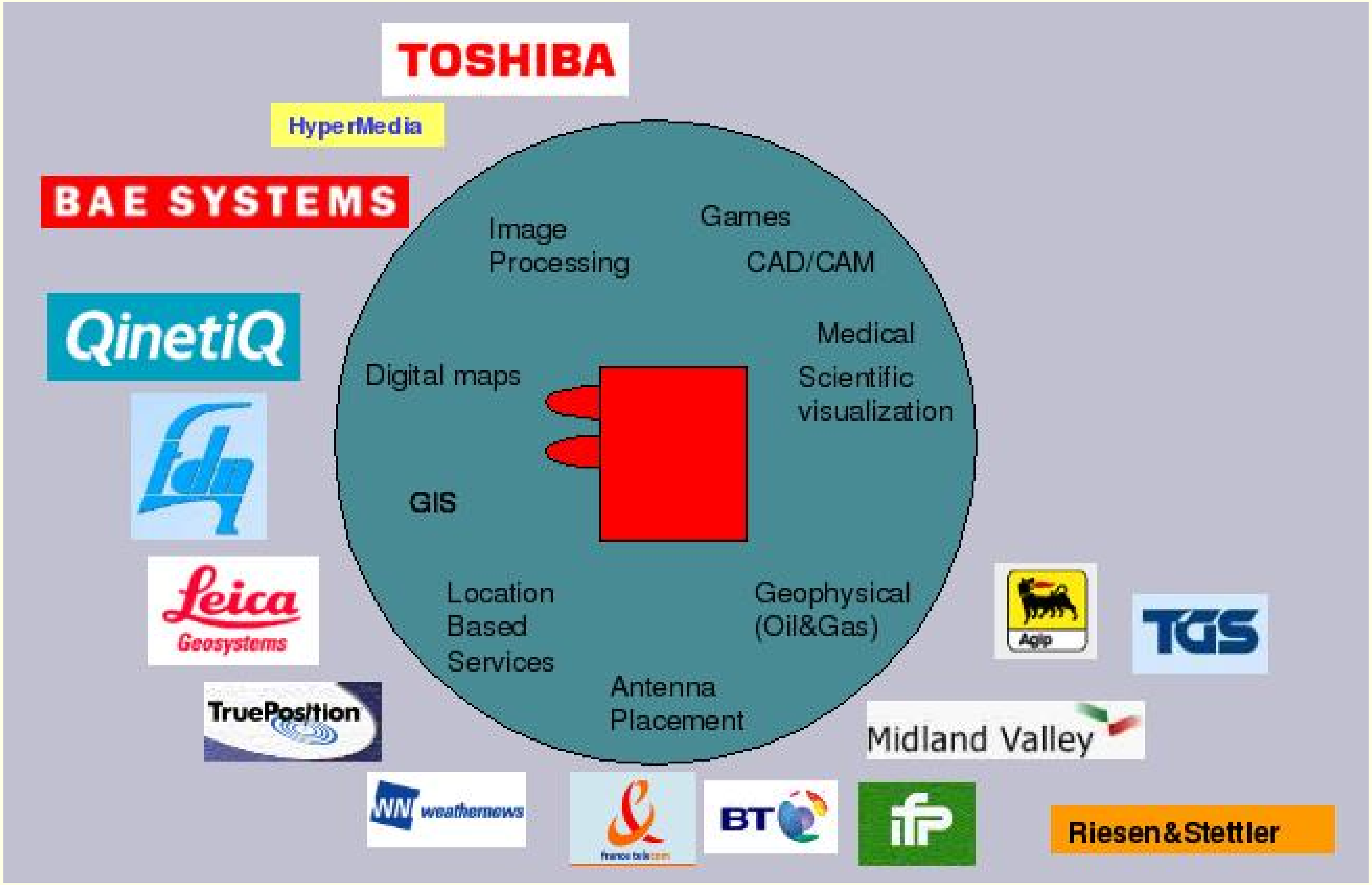
Peter Coveney and co-workers, University of London.

. . .

Teaching

- Leo Guibas, Siu Wing Cheng, . . .

Commercial customers of Geometry Factory



Structure of CGAL

Basic Library

Algorithms and Data Structures

Kernel

Geometric objects
Geometric operations

core library

configurations, assertions, . . .

Support Library

Visualization
File
I/O
NumberTypes
Generators
. . .

The CGAL Kernel

In the kernel

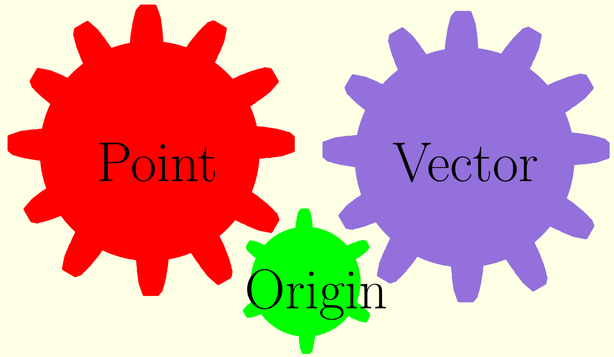
Elementary geometric objects

Elementary computations on them

Primitives **2D, 3D, dD**	**Predicates**	**Constructions**
• Point	• comparison	• intersection
• Vector	• Orientation	• squared distance
• Triangle	• InSphere	. . .
• Iso_rectangle	. . .	
• Circle		
. . .		

Affine geometry

Point - Origin → Vector
Point - Point → Vector
Point + Vector → Point



Point + Point **illegal**

midpoint(a,b) = a + 1/2 x (b-a)

Kernels and Number Types

Cartesian representation

Point $\left| \begin{array}{l} x = \frac{hx}{hw} \\ y = \frac{hy}{hw} \end{array} \right.$

Homogeneous representation

Point $\left| \begin{array}{l} hx \\ hy \\ hw \end{array} \right.$

Intersection of two lines

$$\left\{ \begin{array}{l} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} a_1hx + b_1hy + c_1hw = 0 \\ a_2hx + b_2hy + c_2hw = 0 \end{array} \right.$$

$$(x, y) = \left(\frac{\left| \begin{array}{cc} b_1 & c_1 \\ b_2 & c_2 \end{array} \right|}{\left| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array} \right|}, -\frac{\left| \begin{array}{cc} a_1 & c_1 \\ a_2 & c_2 \end{array} \right|}{\left| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array} \right|} \right)$$

$$(hx, hy, hw) = \left(\left| \begin{array}{cc} b_1 & c_1 \\ b_2 & c_2 \end{array} \right|, -\left| \begin{array}{cc} a_1 & c_1 \\ a_2 & c_2 \end{array} \right|, \left| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array} \right| \right)$$

Field operations

Ring operations

C++ Templates

CGAL::**Cartesian**< **FT** > (CGAL::**Simple_Cartesian**)
CGAL::**Homogeneous**< **RT** > (CGAL::**Simple_Homogeneous**)

- Cartesian Kernels : Field type
  - double
  - Quotient<Gmpz>
  - leda_real
- Homogeneous Kernels : Ring type
  - int
  - Gmpz
  - double

⟶ **Flexibility**

```
typedef double                         NumberType;
typedef Cartesian< NumberType >        Kernel;
typedef Kernel::Point_2                Point;
```

Numerical Issues

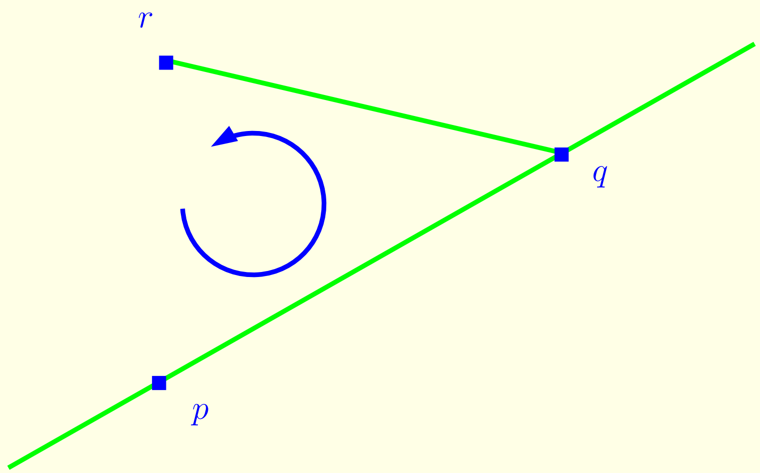
```
typedef CGAL::Cartesian<NT> Kernel;
NT sqrt2 = sqrt( NT(2) );

Kernel::Point_2 p(0,0), q(sqrt2,sqrt2);
Kernel::Circle_2 C(p,2);

assert( C.has_on_boundary(q) );
```

OK if NT gives exact `sqrt`
assertion violation otherwise

Orientation of 2D points



$$
\begin{aligned}
orientation(p,q,r) &= \textbf{sign}\left(\det\begin{bmatrix} p_x & p_y & 1 \\ q_x & q_y & 1 \\ r_x & r_y & 1 \end{bmatrix}\right) \\
&= \textbf{sign}((q_x - p_x)(r_y - p_y) - (q_y - p_y)(r_x - p_x))
\end{aligned}
$$

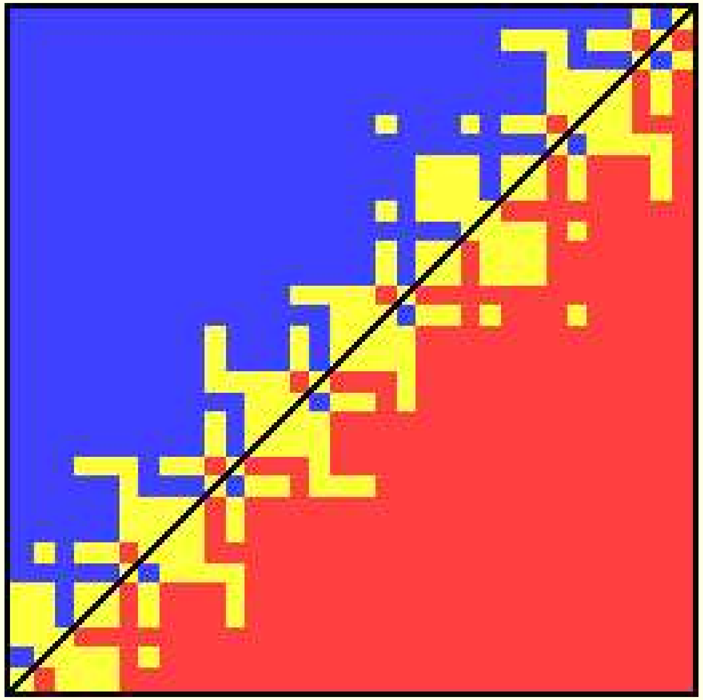
$p = (0.5 + x.u,\ 0.5 + y.u)$
$0 \leq\ x, y\ < 256,\ \ u = 2^{-53}$
$q = (12, 12)$
$r = (24, 24)$

$orientation(p, q, r)$

evaluated with `double`

256 x 256 pixel image

> 0, $= 0$, < 0



$\longrightarrow$ **inconsistencies** in predicate evaluations

[Kettner, Mehlhorn, Pion, Schirra, Yap, ESA'04]

Numerical Robustness in CGAL

imprecise numerical evaluations

$\longrightarrow$ non-robustness

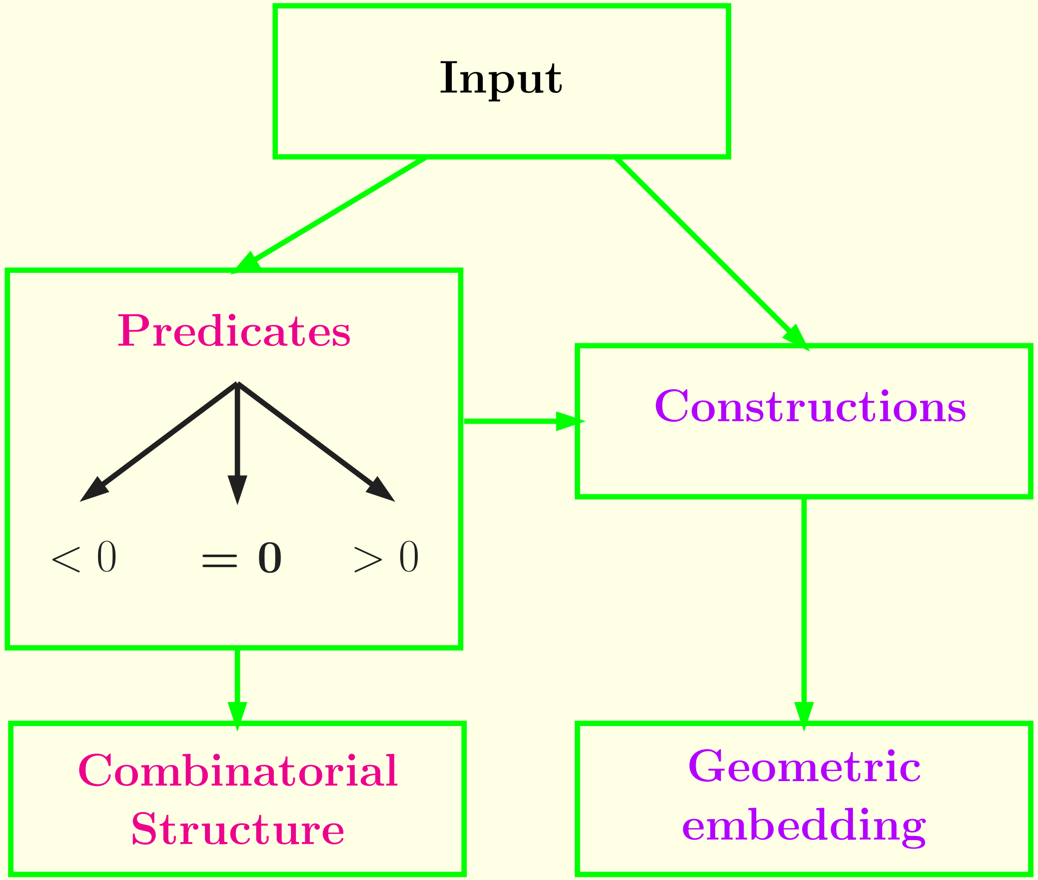
combinatorial result

Exact Geometric Computation

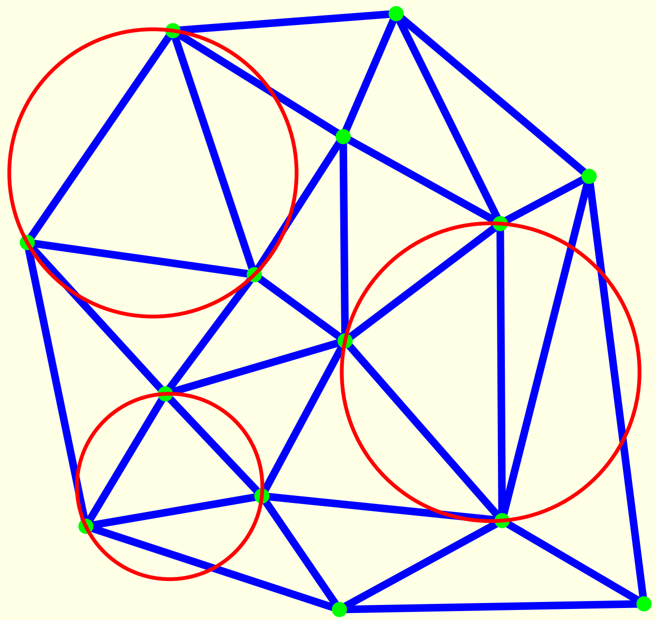
$\neq$

exact arithmetics

Predicates and Constructions

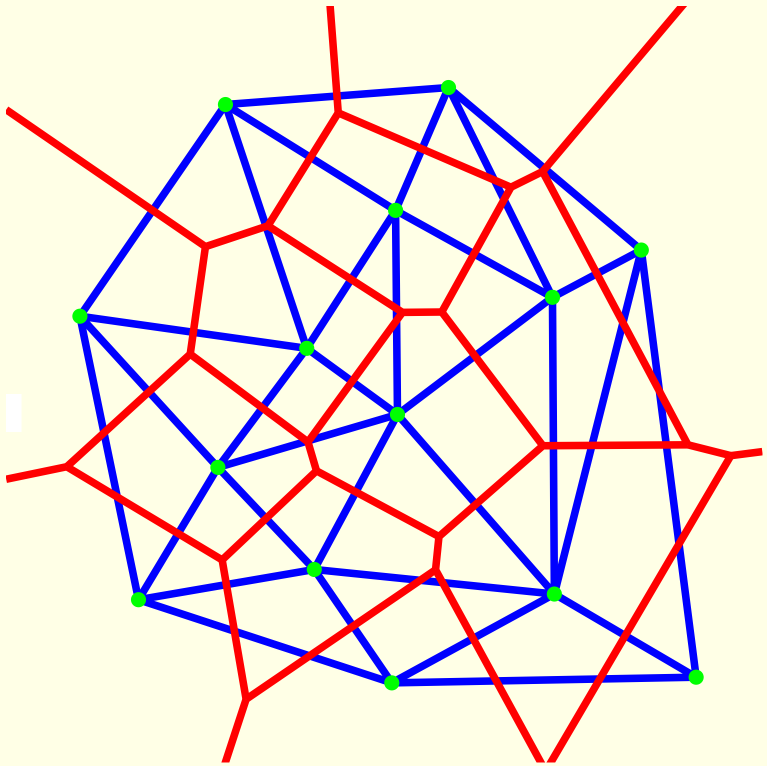


Delaunay triangulation



only **predicates** are used
orientation, in_sphere

Voronoi diagram



constructions are needed
circumcenter

Arithmetic tools

- **Multiprecision integers**
  Exact evaluation of signs / values of polynomial expressions with integer coefficients
  CGAL::MP_Float, GMP::mpz_t, LEDA::integer, ...

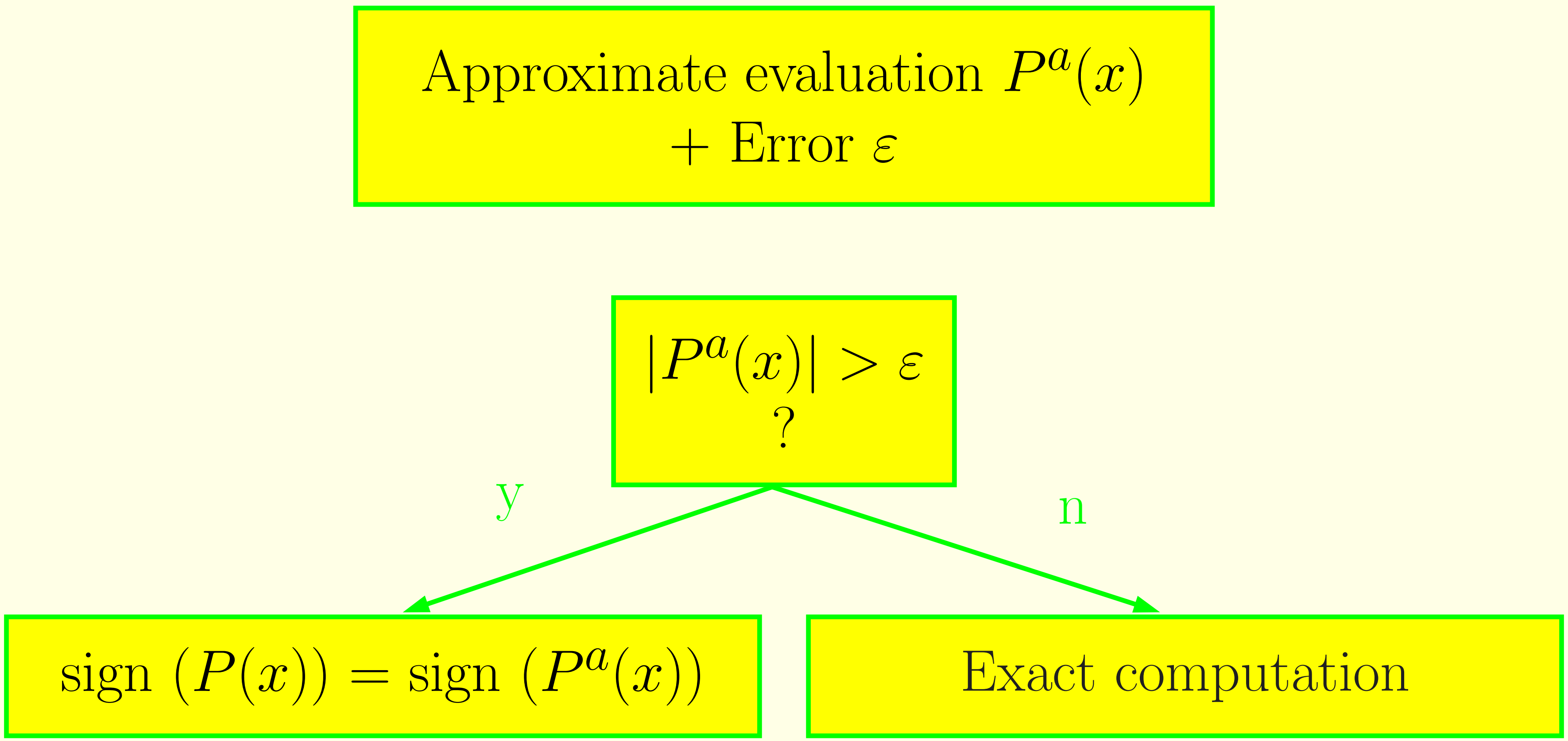
- **Multiprecision floats**
  idem, with float coefficients $(n2^m, n, m \in \mathbb{Z})$
  CGAL::MP_Float, GMP::mpf_t, LEDA::bigfloat, ...

- **Multiprecision rationals**
  Exact evaluation of signs / values of rational expressions
  CGAL::Quotient$< \cdot >$, GMP::mpq_t, LEDA::rational, ...

- **Algebraic numbers**
  Exact comparison of roots of polynomials
  LEDA::real, Core::Expr (work in progress in CGAL)

Filtering Predicates

sign $(P(x))$ **?**



Static filtering

Error bound precomputed — faster

Dynamic filtering

Interval arithmetic — more precise

Number types: **CGAL::Interval_nt, MPFR/MPFI, boost::interval**

CGAL::Filtered_kernel $<$ **K** $>$ kernel wrapper [Pion]

Replaces predicates of **K** by filtered and exact predicates.
(exact predicates computed with MP_Float)

Static + Dynamic filtering in CGAL 3.1

$\longrightarrow$ more generic generator also available for user's predicates

Filtering Constructions

Number type **CGAL::Lazy_exact_nt** $<$ **Exact_NT** $>$ [Pion]

Delays exact evaluation with **Exact_NT**:

- stores a **DAG** of the expression
- computes first an approximation with **Interval_nt**
- allows to control the relative precision of `to_double`

CGAL::Lazy_kernel in CGAL 3.2

Predefined kernels

Exact_predicates_exact_constructions_kernel
Filtered_kernel< Cartesian< Lazy_exact_nt< Quotient< MP_Float >>>>

Exact_predicates_exact_constructions_kernel_with_sqrt
Filtered_kernel< Cartesian< Core::Expr >>

Exact_predicates_inexact_constructions_kernel
Filtered_kernel< Cartesian< double >>

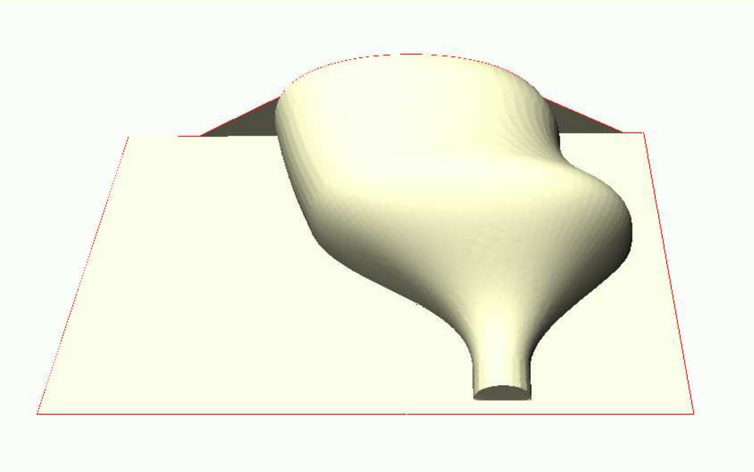
Efficiency

3D Delaunay triangulation
CGAL-3.1-I-124

Pentium-M 1.7 GHz, 1GB
g++ 3.3.2, -O2 -DNDEBUG

1.000.000 random points

Simple_Cartesian$<$ double $>$	**48.1 sec**
Simple_Cartesian$<$ MP_Float $>$	**2980.2 sec**
Filtered_kernel (dynamic filtering)	232.1 sec
Filtered_kernel (static + dynamic filtering)	**58.4 sec**



49.787 points (Dassault Systèmes)

double	**loop !**
exact and filtered	$<$ 8 sec

Robustness of Delaunay triangulations

Kernel and arithmetics $\longrightarrow$ Numerical robustness

Algorithms $\longrightarrow$ explicit treatment of **degenerate cases**

Symbolic perturbation for 3D dynamic Delaunay triangulations
[Devillers Teillaud SODA'03]

Contents of
the CGAL Basic Library

Convex Hull

[MPI]

- 5 different algorithms in 2D
- 3 different algorithms in 3D

Triangulations and related

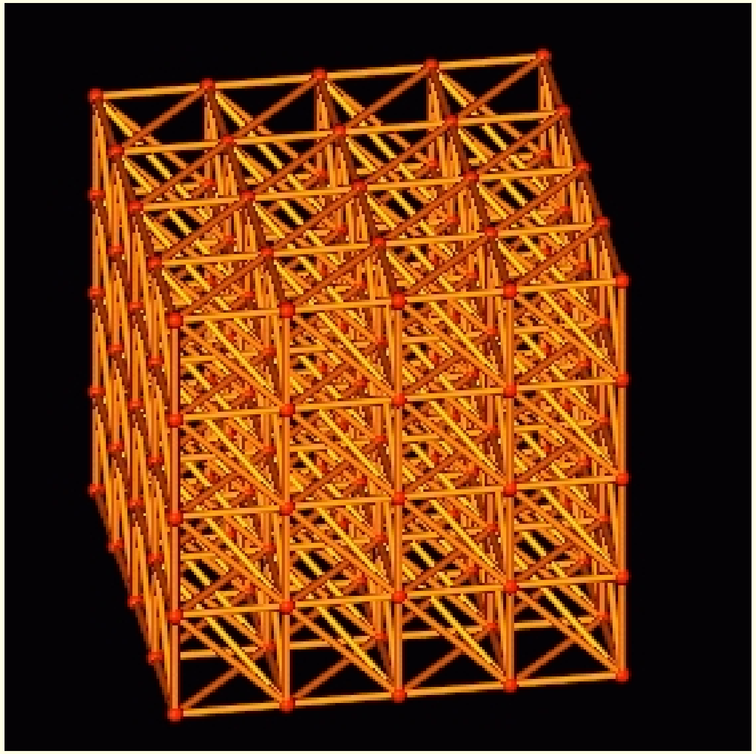
[INRIA]

• 2D/3D Triangle/Tetrahedron based data-structure

• Fully dynamic 2D/3D Delaunay triangulation
  Delaunay hierarchy [Devillers '98 '02]

• 2D/3D Regular Triangulations (fully dynamic in 3.2?)
• 2D Constrained Delaunay Triangulation

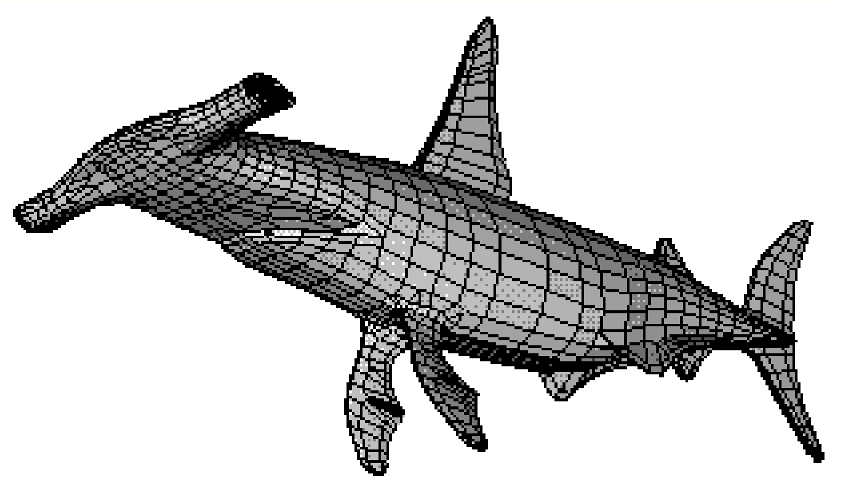
• 2D Apollonius diagram
• 2D Segment Voronoi Diagram
• 2D Meshes



Polyhedra

[MPI]

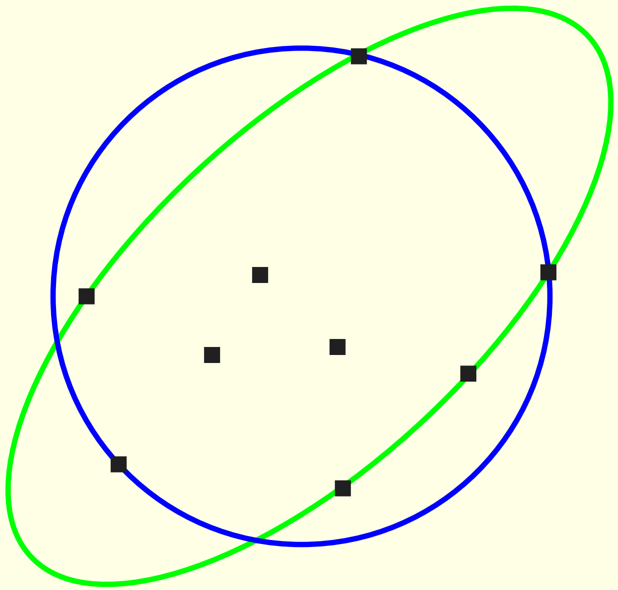
- Half-edge data-structure
- Polyhedral surface
  (orientable 2-manifold with boundary)
- 2D Nef polygons
- 3D Nef polyhedra



Geometric Optimization

[ETH]

- Smallest enclosing circle and ellipse in 2D
- Smallest enclosing sphere in dD
- Largest empty rectangle
- . . .



Arrangements

`[Tel-Aviv]`

- Line segments or polylines
- Conic arcs with Leda or Core

Completely new version in CGAL 3.2

Search Structures

Arbitrary dimension

- Range-tree, Segment-tree, kD-tree
- Window query
- Approximate nearest neighbors
- . . .

Flexibility in the CGAL Basic Library

“Traits” classes

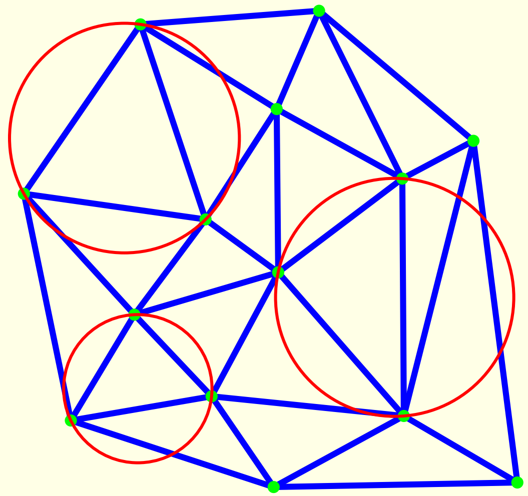
convex_hull_2<InputIterator, OutputIterator, **Traits**>
Polygon_2<**Traits**, Container>
Polyhedron_3<**Traits**, HDS>
Triangulation_3<**Traits**, TDS>
Min_circle_2<**Traits**>
Range_tree_k<**Traits**>
. . .

Geometric traits classes provide:
Geometric objects + predicates + constructors

- The **Kernel** can be used as a traits class for several algorithms
- Otherwise: **Default traits classes** provided
- The **user** can plug his own traits class

Playing with traits classes

Delaunay Triangulation



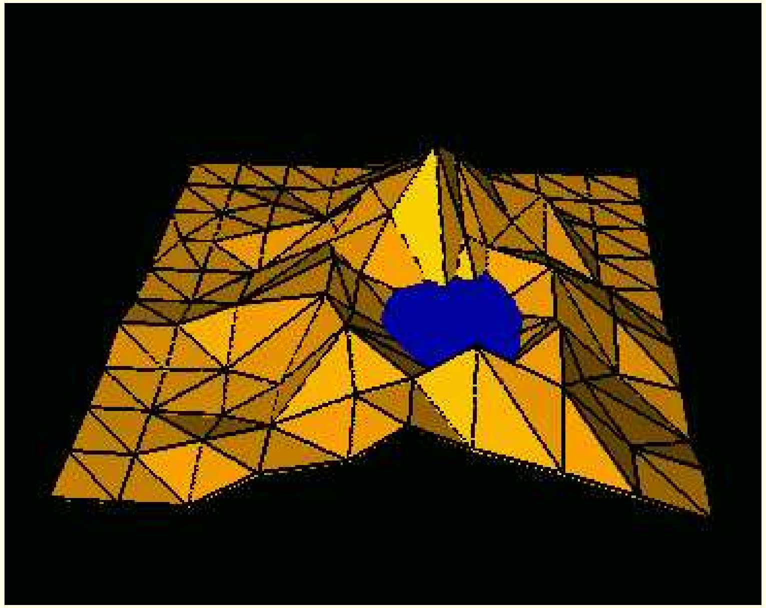
Requirements for a traits class:

- Point
- orientation test, in_circle test

```
typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef CGAL::Delaunay_triangulation_2< K > Delaunay;
```

- 3D points: coordinates $(\mathbf{x}, \mathbf{y}, \mathbf{z})$
- orientation, in_circle: on $\mathbf{x}$ and $\mathbf{y}$ coordinates

```
typedef CGAL::Exact_predicates_inexact_constructions_kernel K;
typedef CGAL::Triangulation_euclidean_traits_xy_3< K > Traits;
typedef CGAL::Delaunay_triangulation_2< Traits > Terrain;
```



More flexibility

The user can add information in vertices and cells

. . .

Work in Progress

Kinetic Data Structures
[Russel Karavelas]

Persistent Homology
[Kettner Zomorodian]

Surface reconstruction
[Oudot Rey]

3D Meshes
[Rineau Yvinec]

Parameterization
[Alliez]

Curved Kernel
Extension of the CGAL kernel
Algebraic issues
[Emiris Kakargias Pion Tsigaridas Teillaud SoCG'04]

...